KIET IJCE KIET International Journal of Communications & Electronics
Volume. No. 6, Issue No. 2, July-Dec., 2018, ISSN: 2320 – 8996

Modal Analysis for a Step Index Fiber

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ABSTRACT

In this paper we have computed the normalized propagation constant on an assumed optical fiber using weakly guide approximation.A Bragg fiber is usually designed to minimize the leakage for some particular mode. Again, we have the problem of leakage due to finite number of claddings. As a result, some higher-order undesired modes may be supported by the same Bragg fiber with larger attenuation coefficients. However, it can be shown numerically that these undesired modes are very lossy .Therefore the Bragg fiber can be employed as a mode filter to select some particular mode from an ensemble of modes.

***Keywords*:** Weak guidance; Dispersion curves; .

INTRODUCTION

An optical waveguide is a structure which confines and guides the light beam by the process of total internal reflection. The most extensively used optical waveguide is the step index optical fiber which consists of a cylindrical central core, clad by a material of slightly lower refractive index. A completely different confinement mechanism, Bragg reflection, provides an alternative way of guiding photons .Bragg fiber consist of a center dielectric core surrounded by cladding layers with alternating low and high refractive indexes.

A Bragg fiber is usually designed to minimize the leakage for some particular mode. Again, we have the problem of leakage due to finite number of claddings. As a result, some higher-order undesired modes may be supported by the same Bragg fiber with larger attenuation coefficients. However, it can be shown numerically that these undesired modes are very lossy .Therefore the Bragg fiber can be employed as a mode filter to select some particular mode from an ensemble of modes.

Modal Analysis for a Step index Fiber

In this section, we will obtain the modal fields and the corresponding propagation constants for a step index fiber for which the refractive index variation is known.

$$n(r) = \begin{cases} n_1 & 0 < r < a \quad \text{core} \\ n_2 & r > a \quad \text{cladding} \end{cases} \qquad (1)$$

In most practical fibers used in communication are weakly guiding i.e. relative index difference is

$$\frac{n_1 - n_2}{n_2} \leq 0.01 \qquad (2)$$

And this allows use of the so-called scalar wave approximation (also known as the weakly guiding approximation). In this approximation, the modes are assumed to be nearly transverse and can have an arbitrary state of polarization. Thus, the two independent sets of modes can be assumed to be x-polarized and y-polarized, and in the weakly guiding approximation they have the same propagation constants. These are usually referred to as LP modes; LP stands for linearly polarized. It's mentioned that when n1 ~ n2, the modes are nearly transverse and the propagation constants of the TE and TM modes are almost equal. In the weakly guiding approximation, the transverse component of the electric field (Ex or Ey) satisfies the scalar wave equation.

$$\nabla^2 \Psi = \varepsilon_0 \mu_0 n^2 \frac{\partial^2 \Psi}{\partial t^2} = \frac{n^2}{c^2} \frac{\partial^2 \Psi}{\partial t^2} \qquad (3)$$

where $c(=1/\sqrt{\varepsilon_0 \mu_0}) \approx 3 \times 10^8 \text{ m s}^{-1}$ is the speed of light in free space. In most practical fibers n2 depends only on the cylindrical coordinate r, and therefore it is convenient to use the cylindrical system of coordinates $(r, \phi, z,)$ and write the solution of Eq. (3) in the form

$$\Psi(r, \phi, z, t) = \psi(r, \phi) e^{i(\omega t - \beta z)} \qquad (4)$$

Where w is the angular frequency and b is known as the propagation constant. The above equation defines the modes of the system. Since $\Psi(r, \phi)$ depends only on the transverse coordinates r and ϕ, ,

The modes represent transverse field configurations that do not change as they propagate through the optical fiber except for a phase change.

In the cylindrical system of coordinates $(r, \phi, z,)$ we have

$$\nabla^2 \Psi = \frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \phi^2} + \frac{\partial^2 \Psi}{\partial z^2} \qquad (5)$$

Now, from Eq. (4) it readily follows that

$$\frac{\partial^2 \Psi}{\partial t^2} = -\omega^2 \Psi = -\omega^2 \psi(r, \phi) e^{i(\omega t - \beta z)} \qquad (6)$$

And

$$\frac{\partial^2 \Psi}{\partial z^2} = -\beta^2 \Psi = -\beta^2 \psi(r, \phi) e^{i(\omega t - \beta z)} \qquad (7)$$

Substituting Eq. (4) in Eq. (3) and using Eqs. (5) to (7), we obtain

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\phi^2}+[k_0^2n^2(r)-\beta^2]\psi=0 \quad \textbf{(8)}$$

Where,

$$k_0=\frac{\omega}{c}=\frac{2\pi}{\lambda_0}$$

is the free space wave number. Because the medium has cylindrical symmetry, i.e., n^2 depends only on the cylindrical coordinate r, we can solve Eq. (8) by the method of separation of variables:

$$\psi(r,\phi)=R(r)\,\Phi(\phi)$$

On substituting and dividing by $\psi(r,\phi)/r^2$, we obtain

$$\frac{r^2}{R}\left(\frac{d^2R}{dr^2}+\frac{1}{r}\frac{dR}{dr}\right)+r^2\left[n^2(r)k_0^2-\beta^2\right]=-\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=+l^2 \quad \textbf{(9)}$$

Thus the variables have separated out, and we have set each side equal to a constant $(=l^2)$. Solving the equation depending only on Φ, we find that the Φ dependence will be of the form $\cos l\phi$ or $\sin l\phi$, and for the function to be single-valued [i.e., for $\Phi(\phi+2\pi)=\Phi(\phi)$] we must have *l*= 0, 1, 2, . . .

Negative values of l correspond to the same field distribution. Thus the complete transverse field is given by

$$\Psi(r,\phi,z,t)=R(r)e^{i(\omega t-\beta z)}\begin{Bmatrix}\cos l\phi\\ \sin l\phi\end{Bmatrix}\; l=0,1,2,\ldots \quad \textbf{(10)}$$

Where R(r) satisfied the radial part of the equation

$$r^2\frac{d^2R}{dr^2}+r\frac{dR}{dr}+\left\{\left[k_0^2n^2(r)-\beta^2\right]r^2-l^2\right\}R=0$$

Equation (11) is an eigenvalue equation with β^2 representing the eigenvalue. By applying the appropriate boundary conditions, we will show that β^2 can have a set of discrete values (corresponding to guided modes of the waveguide) and also a continuum of values corresponding to radiation modes of the waveguide.

Since for each value of *l* there can be two independent states of polarization, modes with *l*≥ 1 are fourfold degenerate (corresponding to two orthogonal polarization states and to the Φ dependence being $\cos l\phi$ or $\sin l\phi$). Modes with l = 0 are Φ independent and have twofold degeneracy. We cannot set the right-hand side of Eq. (9) equal to a negative constant, because then the Φ dependence of the field will not be single-valued. In the next section we give the solution of Eq. (11) forastepindexprofile. However, foranarbitrarycylindricallysymmetricprofilehavingarefractiveindex thatdecreasesmonotonicallyfromavaluen[1]ontheaxistoaconstantvalue n[2]beyondthecore-claddinginterface*r* = *a*,wecanmakethegeneralobservation that hesolutionsofEq. (11) canbedividedintotwodistinctclasses. Thefirstclassofsolutionscorrespondsto

$$n_2^2<\frac{\beta^2}{k_0^2}<n_1^2 \qquad \text{guided modes} \qquad \textbf{(12)}$$

For β^2 lyingintheaboverange, thefieldsR(r) areoscillatoryinthecoreanddecay in the claddingand β^2 assumeonlydiscretevalues; theseareknownastheguidedmodesofthewaveguide. Foragivenvalueof*l*, therewillbeafinitenumberofguidedmodes, thesearedesignatedasLP$_{lm}$modes (*m* = 1, 2, 3, . . .). Theseclassofsolutionscorrespondsto

$$\beta^2<k_0^2n_2^2 \qquad \text{radiation modes} \qquad \textbf{(13)}$$

Forsuch β values, thefieldsareoscillatoryeveninthecladdingand β canassumeacontinuumofvalues. Theseareknown.

Guided Modes of a Step index Fiber

Forsuchafiber, forguidedmodes (forwhich $n_2^2<\beta^2/k_0^2<n_1^2$), Eq. (11) canbewrittenintheform

$$r^2\frac{d^2R}{dr^2}+r\frac{dR}{dr}+\left(U^2\frac{r^2}{a^2}-l^2\right)R=0 \qquad 0<r<a \quad \textbf{(14)}$$

And

$$r^2\frac{d^2R}{dr^2}+r\frac{dR}{dr}-\left(W^2\frac{r^2}{a^2}+l^2\right)R=0 \qquad r>a \quad \textbf{(15)}$$

Where,

$$U\equiv a\sqrt{k_0^2n_1^2-\beta^2} \quad \textbf{(16)}$$

And

$$W\equiv a\sqrt{\beta^2-k_0^2n_2^2} \quad \textbf{(17)}$$

Because of Eq. (12), both *U* and *W* are real. The normalized waveguide parameter *V* is defined by

$$V = \sqrt{U^2 + W^2} = k_0 a \sqrt{n_1^2 - n_2^2} \qquad \textbf{(18)}$$

In terms of the wavelength

$$V = \frac{2\pi}{\lambda_0} a \sqrt{n_1^2 - n_2^2} \qquad \textbf{(19)}$$

The waveguide parameter V is an extremely important quantity characterizing an optical fiber. It is convenient to define the normalized propagation constant

$$b = \frac{\beta^2/k_0^2 - n_2^2}{n_1^2 - n_2^2} = \frac{W^2}{V^2} \qquad \textbf{(20)}$$

Thus

$$W = V\sqrt{b} \qquad \textbf{(21)}$$

and

$$U = V\sqrt{1-b} \qquad \textbf{(22)}$$

From Eq. (12) we find that for guided modes 0 < b < 1. The two independent solutions of Eq. (14) are $J_l(Ur/a)$ and $Y_l(Ur/a)$; however, the solution $Y_l(Ur/a)$ has to be rejected since it diverges as $r \rightarrow 0$. The solutions of Eq. (15) are the modified Bessel functions $K_l(Wr/a)$ and Il(Wr/a); the solution Il(Wr/a) has to be rejected since it diverges as $r \rightarrow \infty$. Thus, for guided modes, the transverse dependence of the modal field is given by

$$\psi(r,\phi) = \begin{cases} \dfrac{A}{J_l(U)} J_l\left(\dfrac{Ur}{a}\right)\begin{bmatrix}\cos l\phi \\ \sin l\phi\end{bmatrix} & r < a \\ \dfrac{A}{K_l(W)} K_l\left(\dfrac{Wr}{a}\right)\begin{bmatrix}\cos l\phi \\ \sin l\phi\end{bmatrix} & r > a \end{cases} \qquad \textbf{(23)}$$

where A is a constant and we have assumed the continuity of ψ at the core-cladding interface $(r = a)$. Continuity of $\partial\psi/\partial r$ at $r = a$ and use of identities involving Bessel functions give the following transcendental equations which determine the allowed discrete values of the normalized propagation constant b of the guided LP_{lm} modes:

$$V\sqrt{1-b}\,\frac{J_{l-1}\left[V\sqrt{1-b}\right]}{J_l\left[V\sqrt{1-b}\right]} = -V\sqrt{b}\,\frac{K_{l-1}\left[V\sqrt{b}\right]}{K_l\left[V\sqrt{b}\right]};\ l \geq 1 \qquad \textbf{(2}$$

and

$$V\sqrt{1-b}\,\frac{J_1\left[V\sqrt{1-b}\right]}{J_0\left[V\sqrt{1-b}\right]} = V\sqrt{b}\,\frac{K_1\left[V\sqrt{b}\right]}{K_0\left[V\sqrt{b}\right]};\ l = 0$$

Thesolutionoftheabovetranscendentalequationswillgiveuniversalcurvesdescribingthedependenceof*b* (andthereforeof*U*and*W*) on*V*. For a givenvalueof *l*, therewillbeafinitenumberofsolutions, andthesolution

(*m* = 1, 2, 3, . . .)isreferredtoastheLP$_{lm}$mode. The variationofwithVformofuniversalcurves, forvaluesofVlyingbetween1.0and10

RESULT

In this section, we use the above method to analyze the modal dispersion of an air core Bragg fiber. The eigen value Equations has all the information that we can obtain from our modal analysis and it gives the central results of this investigation. It is convenient to plot the normalized propagation constant

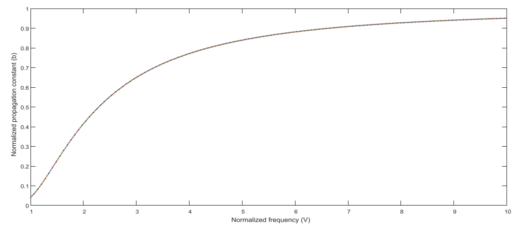

Fig. 2. Dispersion curves of normalized frequency V versus normalized propagation constant ' b

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