

# Review on Theory of Non Uniform waveguides

Reema Budhiraja<sup>1</sup>, Manish kumar<sup>2</sup>, R C Jain<sup>3</sup>

JIIT, Sector-62, Noida

[reema.budhiraja@jiit.ac.in](mailto:reema.budhiraja@jiit.ac.in), [manish.kumar@jiit.ac.in](mailto:manish.kumar@jiit.ac.in), [rc.jain@jiit.ac.in](mailto:rc.jain@jiit.ac.in)

**Abstract**— Uniform waveguides are widely used in telecommunications systems both for ground and space applications. The theory of uniform waveguides is well-known but not all waveguides of a telecommunication system are uniform and so-called non-uniform waveguides are also encountered. This paper reviews various kinds of non uniformity observed in waveguides and different methods adopted by researchers for analysing these non-uniform waveguides have been discussed.

**Keywords**— Non-Uniform, Waveguides, Taper, Coupling Coefficient, Orthogonal

## I. INTRODUCTION

Non-uniform element is a gradual waveguide taper with a length which is longer than either of the dimensions of the waveguide cross section. The parameters of these waveguides are slowly changing functions of one of the coordinates. A bent waveguide with a radius of curvature larger than the cross section dimensions or a long tapering between two waveguides with different cross sections or a long waveguide twist comes under the category of non-uniform waveguides. These devices operate in a broad frequency band. Since the cross sections of these waveguides is essentially larger than the wavelength, so lot of various modes can propagate in gradual waveguide transitions. These waveguides are designed in such a way that the main part of the incident power is carried away by one specific wave while simultaneously the amplitudes of the other modes are maintained very small.

Any complicated waveguide non-uniformity can be considered as the superposition of several basic kinds of non-uniformity like: a bend, a variation of the filling medium, a variation of the wall impedance and a variation of the waveguide cross-section. A waveguide bent by a finite angle can be considered as the limit of a waveguide consisting of a lot of small tilts. Similarly, the properties of a continuously varying filling medium can be represented by the limit of a waveguide filled with a multilayered medium, and a variable cross section waveguide is the limit of a lot of small cross-section steps. Now different methods adopted by the

researchers for analysing these non-uniform waveguides have been discussed.

## II. METHOD OF FIELD CONTINUITY ADJUSTMENT

The method of field continuity adjustment is based on field expansion in to a sum of waves forming the electromagnetic field of the uniform and non-uniform waveguide and on the continuity requirements for the fields at the boundary which leads to linear algebraic coupled wave equations for the coefficients of these expansions. The application of this method is limited to simple non-uniform waveguide structures. In the works of Lewin (1955a and b), Piefke (1957) and Solymar (1959b) the reflection Coefficients of the wave were obtained by this method for rectangular waveguide broadening [1]. An equivalent method was employed by Mar' in to determine the coupling and reflection coefficients when a rectangular waveguide is broadened in the E-plane. Solymar (1958, 1959d) applied this method to circular waveguides for the determination of the coupling coefficient between the TE<sub>01</sub> and TE<sub>02</sub> modes [2] [3][4]. Also, Tanaka (1957) did the same for the calculation of the coupling and reflection coefficients in the general case of arbitrary wave incidence [5]. Jouget (1947a) also solved the problem of the interconnection of two rectangular waveguides, one being straight and the other being bent, by the method of field continuity adjustment. But this method is limited to simple non uniform waveguide structures.

## III. CONFORMAL MAPPING

Conformal mapping was used by Krasnushkin (1945) for the analysis of planar waveguides [6]. By means of this method the complicated boundary of the non-uniform waveguide is transformed at the limit in to two straight parallel lines. After which the wave equation describing the field in the waveguide becomes more complicated and adopts a form equivalent to an inhomogeneous medium placed between two parallel lines. The parameters of this inhomogeneity are related to the conformal mapping function. Rozhdestvenskiy and Chetayev(1951) used conformal mapping to solve the problem of matching transitions with dielectric filling [7]. Conformal mapping was also applied by Weinstein (1957) where the



problem of a slow varying non-uniformity in a planar waveguide was solved by variational methods. Now conformal mapping can be used for the case of circular straight waveguides but it can not be employed for more complex problems like the simultaneous broadening of a rectangular waveguide in two planes, so for these cases special methods were used.

#### **IV. MATCHED CO-ORDINATE SYSTEM**

In this method a coordinate system was introduced where Maxwell's equations incorporate additional terms proportional to the curvature, in contrast to the Cartesian orthogonal system. These terms represent supplementary currents generated by the propagating wave. This method was used by Jouget (1947b) for circular waveguide, bent along a circumference with a large radius  $r$ . The publication by Lewin (1955b) also relates to this method, where the wave numbers of the eigen waves for twisted and curved rectangular waveguides are determined. Viktor ova and Sveshnikov (1958) also applied this method for a waveguide bent along a double curvature line and simultaneously having a slow variation of its cross section [8]. Pokrovskii, Ulinich and Savvinyk (1958) proposed a productive idea for the calculation of straight waveguides with variable cross section, based on the introduction of a special system of coordinates in their paper about planar waveguides [9]. Later on Discontinuous horn transitions between two waveguides, that means waveguides with their generatrices described by analytical functions having discontinuities in the first derivative, and problems relating to cut-off cross section waveguides were investigated using this method.

#### **V. CROSS SECTION METHOD**

In this method the electromagnetic field in a nonuniform waveguide is represented by means of a superposition of the mode fields corresponding to more simple waveguides. The coefficients of this superposition satisfy ordinary differential coupled wave equations and from the solution of these coupled wave equations the amplitudes of the waves scattered by the non uniform waveguide can be determined. Stevenson (1951a) discussed straight waveguides with variable cross section in which the electromagnetic field was expressed through six functions, each of which was expanded in terms of eigenfunctions of TE and TM modes corresponding to the constant cross-section uniform waveguide [10]. Now to obtain the coefficients of these expansions, second order differential coupled wave equations were derived. But the mathematical apparatus obtained from this procedure is very large and complicated and the only attempt to apply it to practical

problems for the determination of the field scattered on a nonuniform waveguide section was done by Leonard and yen (1957) [11]. In this work they calculated the reflection coefficients for several waves at the connection of a straight circular waveguide with a cone and moreover the reflections from the broadening of a rectangular waveguide. The formulae obtained for the rectangular waveguide are still valid, but for the case of circular waveguides they are wrong

The cross section method is proposed again by Schelkunoff (1955) and it is illustrated by several examples of broadening planar waveguides and also by waveguides bent along a circumference arc [12]. The work published by Heyn (1955) also presents the expansion of the wave fields of a constant cross-section bent waveguide through straight waveguide fields. In both papers, first order differential equations were established for the determination of the expansion coefficients, but these equations remain unsolved and the expressions for the scattered wave amplitudes were not presented.

Unger then in (1958a) applies the mathematical apparatus proposed by Schelkunoff to the practical problem of the incidence of a TE<sub>01</sub> wave into a symmetrical transition between two circular waveguides [13]. The amplitudes of the waves propagating in both directions are used as variables instead of the coefficients of the field Fourier expansion as it was done in Schelkunoff (1955) and Heyn (1955) and therefore the expressions for the amplitudes of the TE<sub>0n</sub> waves scattered forwards have been obtained.

Reiter (1959) independently from Schelkunoff but employing approximately the same method, studied variable cross section straight waveguides [14]. His results were applied later to the calculation of practical cases in Solymar (1959a) in a more comfortable form. The paper of Lyubarski and Povzner (1957) were also viewed as another version of the cross section method [15].

Variable cross-section straight waveguides were also considered in the papers of Gutman (1957, 1958, 1959a, 1959b). Their originality, compared with Schelkunoff (1955b) and Reiter (1959) and to the further works based on these publications, consists in the introduction of a special system of coordinates as well as in another method to deduce the differential equations. The paper published by Emelin (1958) generalizes this method to the case of simultaneously varying shape of the cross section and the direction of the axis [16].

#### **VI. CONCLUSION**

Electronic devices generating high power millimeter waves produce electromagnetic fields with a very complex structure.



It is necessary to launch this power with minimal losses and with a specific field structure. This makes the use of nonuniform waveguides compulsive. Depending upon the type of nonuniformity, appropriate method is applied for its analysis.

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