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An Improved W-CMSR for Direction Of Arrival Estimation

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***Abstract*—In this paper, an improved W-CMSR method for direction-of-arrival estimation for wideband signal is proposed. An important parameter in direction of arrival estimation by W-CMSR is the fitting error threshold. The accuracy and the angular resolution for direction of arrival estimation hugely depends on this fitting error threshold. This paper propose a method to calculate the fitting error threshold in a way that angular resolution for estimation as well as accuracy of estimation improves. Also the complexity of the proposed method is lesser than the conventional W-CMSR method.**

Keywords—Direction-of-arrival estimation, sensor array, wideband signals, W-CMSR

I. INTRODUCTION

Direction-of-arrival (DOA) denotes the direction from which an unknown incoming wave impinges on the sensor array. The basic device for the direction-of-arrival estimation is the sensor arrays. These sensor arrays are formed by spatially placing the multiple sensor in a desired manner as per the requirement. The spatial arrangement can be linearly, circularly or any other manner in a uniform or non-uniform way as per the requirement.

The direction-of-arrival estimation is an important research area as many application like finding sound source, multi user wireless communication, localizing target by radar, sonar and many more require the accurate information about the direction of the incoming propagating wave. Many techniques has been proposed in past to estimate the direction-of-arrival estimation of the wideband signals such as Music method [1], L1-SVD [2], JLZA [3] and so forth. Most of those method requires the spectral decomposition of the incident signal to estimate the direction of the incoming signals. Method of these category has disadvantage that these method require pre-estimate of the incident signals for the spectral focusing and the accuracy of such pre-estimates largely effects the performance of the system in terms of accuracy and angular resolution. Such method also require the a priori information about the number of signal impinging on the sensor array. Practically, these information are not available for direction-of-arrival estimation. The method of W-CMSR [4] does not require the spectral decomposition of the incoming wideband signals into narrowband signals [5] as well as it does not require any priori information about the number of incoming signals. These advantages of W-CMSR over other method made it popular for direction-of-arrival estimation. In W-CMSR, the lower half element of the covariance matrix are aligned to form a measurement vector. This measurement vector is reconstructed over complete dictionary to perform the direction-of-arrival estimation. The a priori information about the number of sources is not required in this kind of representation and hence this method has a certain advantage over the other method. Also such representation do not break down the incoming wideband signals into narrowband signal and evaluate the incoming signals integrated and therefore reduces the complexity in the estimation process. The restriction for the half wavelength for avoiding uncertainty is eased to lower frequency from higher frequency for this method. This method has the ability to estimate direction-of-arrival of more number of signals than sensors for a well-designed geometry. This paper improves the method of W-CMSR by modifying the fitting error threshold for the direction- of-arrival estimation.

This paper consist of following sections. Section II presents the basic wideband direction-of-arrival model. Section III presents the model of W-CMSR. Section IV discuss the proposed method whereas simulation results are demonstrated in section V. The final conclusions are made in section VI. Section VII consists of the references.

II. PROBLEM FORMULATION

Consider that W sources are transmitting the wideband signals from the direction Φ_1, Φ_2 ... Φ_w. These signal impinges on the array of sensors consisting of N sensors. The output at each sensor is given by

$$x_n(t) = \sum_{w=1}^{W} s_w(t + \tau_{n,w}) + \sigma_n(t) \qquad (1)$$

where σ_n is the additive noise at the n^{th} sensor and s_w is the w^{th} signal arriving at the n^{th} sensor with the propagation delay of $\tau_{n,w}$.

Assuming that Q snapshots are collected, the output of sensors array at time t is given by

$x(t) =$

$$\left[\sum_{w=1}^{W} s_w(t+\tau_{1,w})+\sigma_1(t),\ldots,\sum_{w=1}^{W} s_w(t+\tau_{n,w})+\sigma_n(t)\right]^T \quad (2)$$

Since the wideband signals have the substantial bandwidth, hence the propagation delay of incoming signal in (1) cannot be converted into phase delay.

III. W-CMSR Method

The direction-of-arrival of the wideband signal presented in this section is based on the correlation functions of wideband signals. This correlation function can be extracted from the covariance matrix of the array output.

Assuming the unified correlation function of the incoming signals to be analogous, i.e.

$$r_{w_1}(\tau) = r_{w_2}(\tau)\forall w_1, w_2 \in [1,\ldots. W] \quad (3)$$

The perturbation free covariance matrix is given by

$$C = \begin{bmatrix} \sum_{w=1}^{W} \rho_w + \eta_\sigma^2 & \sum_{w=1}^{W} \rho_w r^*(\tau_{2,w}-\tau_{1,w}) & \cdots & \sum_{w=1}^{W} \rho_w r^*(\tau_{N,w}-\tau_{1,w}) \\ \sum_{w=1}^{W} \rho_w r(\tau_{2,w}-\tau_{1,w}) & \sum_{w=1}^{W} \rho_w + \eta_\sigma^2 & \cdots & \sum_{w=1}^{W} \rho_w r^*(\tau_{N,w}-\tau_{2,w}) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{w=1}^{W} \rho_w r(\tau_{N,w}-\tau_{1,w}) & \sum_{w=1}^{W} \rho_w r(\tau_{N,w}-\tau_{2,w}) & \cdots & \sum_{w=1}^{W} \rho_w + \eta_\sigma^2 \end{bmatrix} \quad (4)$$

where ρ_w is the power of w^{th} signal and η_σ^2 is the variance of additive noise.

The direction-of-arrival of the incoming signals can be estimated from the correlation function family of $r(\tau_{i,w}-\tau_{j,w})$ for each w where $i,j = 1,2,\ldots,N$ but each element of the covariance matrix contains W correlation and diagonal element of the covariance matrix is polluted by the noise variance which is unknown and hence estimation of direction-of-arrival cannot be performed from covariance matrix directly.

It is clear from the (4) that elements above and below the main diagonal of covariance matrix are conjugate to each other and therefore only lower left triangular element is enough to represent the covariance matrix.

Since the diagonal elements are polluted by noise, hence aligning lower left elements column-by-column to form a new one-dimension measurement vector m given by

$m =$

$$\left[C_{2,1},\ldots,C_{N,1},C_{3,2},\ldots,C_{N,2},\ldots,C_{N,N-2},C_{N,N-1}\right]^T \quad (5)$$

where $C_{n1,n2}$ represents $(n_1,n_2)^{th}$ element of Covariance matrix, C.

This vector can also be written as

$$m = \sum_{w=1}^{W} \rho_w k_w \quad (6)$$

where

$k_w =$

$$\left[r(\tau_{2,w}-\tau_{1,w}),\ldots,r(\tau_{N,w}-\tau_{1,w}),\ldots,r(\tau_{N,w}-\tau_{N-1,w})\right] \quad (7)$$

The incoming signal elements coming from the certain directions depends on the unified correlation function. Thus, if the signal elements can be separated then the signal directions can be estimated from m. Considering $\tau_n^{(\phi)}$ be the propagation delay of incoming signal from direction ϕ from reference point to n^{th} sensor then signal elements with unit power is given by

$m^{(\phi)} =$

$$\left[r(\tau_2^{(\phi)}-\tau_1^{(\phi)}),\ldots,r(\tau_N^{(\phi)}-\tau_1^{(\phi)}),\ldots,r(\tau_N^{(\phi)}-\tau_{N-1}^{(\phi)})\right]^T \quad (8)$$

Now, consider the angular grid for incident signal to generate the direction set α. For e.g., if we consider the interval of δ for the angular grid between -90^o to $+90^o$ then the direction set is given by

$$\alpha = \left[-90^0, -90^0+\delta,\ldots,+90^0\right] \quad (9)$$

Thus m can be formulated over complete angular grid as

$$m = m^{(\alpha)}\rho \quad (10)$$

where ρ is the sparse vector having non-zero values. ρ_w (w = 0,1,….,W) indexed matching to the signal direction location in α.

The solution of (10) imposing sparsity is given by

$$\hat{\rho} = \arg\min_{\rho} \|\rho\|_0, \text{ subject to } m = m^{(\alpha)}\rho \quad (11)$$

where $\hat{\rho}$ denotes the spatial distribution of the incident signals and $\| \|_0$ indicates the L0-norm [6].

From (8), it is evident that dictionary elements depends on the correlation function [7] of the incident signals and hence we will first calculate the correlation function.

The correlation function is given by

$$r(\tau) = \frac{1}{P_I}\int_\sigma P(\omega)e^{j\omega\tau}d\omega \quad (12)$$

where $P_I = \int_\sigma P(\omega)d\omega$ and the integral scope σ is fixed according to bandwidth of signal. $P(\omega)$ denotes the signal power spectrum. Output of the sensor array can be used to estimate the power spectrum of signal.

The above model in (11) does not consider the agitations in the practical environment. Considering the

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agitations in the practical environment, we convert L0-norm based problem in (11) into L1-norm optimization problem.

$$\hat{\rho} = \arg\min_{\rho} \| \rho \|_1 \text{, subject to} \| m - m^{(\alpha)} \rho \|_2 \leq \beta \tag{13}$$

where fitting error, β is given by [8]

$$\beta = \mu \times \left\{ \frac{N(N-1)}{2Q} \left[\Xi \left(\sum_{w=1}^{W} \rho_w \right)^2 + 2\eta_\sigma^2 \left(\sum_{w=1}^{W} \rho_w \right) + \eta_\sigma^4 \right] \right\}^{\frac{1}{2}} \tag{14}$$

where μ is the weighting factor and Ξ is given by

$$\Xi = \sum_{\Delta t = qT_s} \left| r(\Delta t) \right|^2 \tag{15}$$

To test the performance of the W-CMSR, we consider two BPSK signals arriving at the sensor array from different angle.

During the experiment, we consider 7 sensor uniform linear array (ULA) with half wavelength spacing between the sensor elements.

Fig. 1., Fig. 2., Fig. 3. and Fig. 4. shows that the W-CMSR method is able to detect the incoming signal. In the experiment, two signals are taken with direction of first signal is fixed to 20^o and the direction of second signal is varied from 45^0 to 30^o with difference of 5^0 between them.

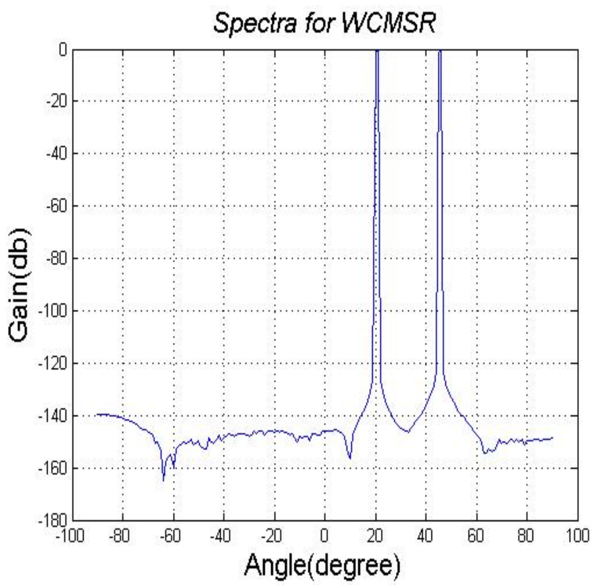

Fig. 1. Spectra of W-CMSR when DOA of signals are 20^0 and 45^0

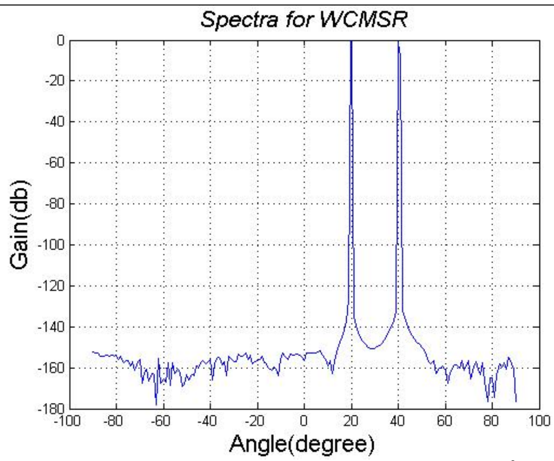

Fig. 2. Spectra of W-CMSR when DOA of signals are 20^0 and 40^0

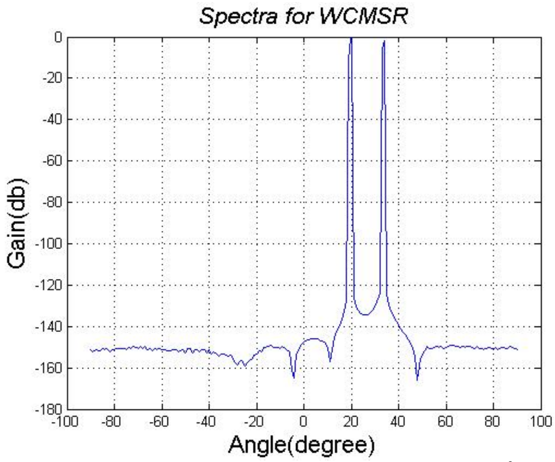

Fig. 3. Spectra of W-CMSR when DOA of signals are 20^0 and 35^0

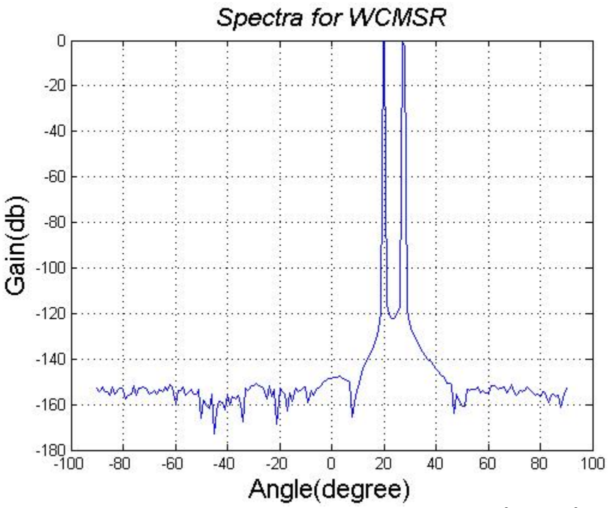

Fig. 4. Spectra of W-CMSR when DOA of signals are 20^0 and 30^0

IV. PROPOSED METHOD

The calculation of fitting error plays the vital role in the accuracy and angular resolution of direction-of-arrival estimation. In this paper, we propose a method to calculate the fitting error in a manner so that accuracy as well as the angular resolution of the W-CMSR method improves. The proper selection of fitting error has the huge impact on feasibility and efficiency of (13).

In this method, we calculate the covariance matrix by two method and then align them to get the two correlation function. The absolute difference between these correlation function is called as residual error. This residual error is used to calculate the fitting error threshold.

The first covariance matrix is calculated as follows

$$R = \frac{1}{Q} * X * X^T \qquad (16)$$

where X is given by

$$X = \left[\sum_{w=1}^{W} s_w(t+\tau_{1,w})+\sigma_1(t), \ldots, \sum_{w=1}^{W} s_w(t+\tau_{n,w})+\sigma_n(t) \right]^T_{t=1\ldots\ldots Q} \qquad (17)$$

The matrix X is called the array output matrix which contains Q snapshots of the output of sensor arrays. Now we align the lower half triangular element of matrix *R* to obtain a new measurement vector M_1.

$$M_1 = \left[R_{2,1}, \ldots, R_{N,1}, R_{3,2}, \ldots, R_{N,2}, \ldots, R_{N,N-2}, R_{N,N-1} \right]^T \qquad (18)$$

The mean of each row of matrix X gives the approximate value of sensor output. Let the mean matrix be denoted by Y and thereby second covariance matrix is calculated as follows

$$S = \frac{1}{Q} * Y * Y^T \qquad (19)$$

Aligning the lower half triangular element of matrix *S* to obtain a measurement vector M_2.

$$M_2 = \left[S_{2,1}, \ldots, S_{N,1}, S_{3,2}, \ldots, S_{N,2}, \ldots, S_{N,N-2}, S_{N,N-1} \right]^T \qquad (20)$$

The absolute difference of M_1 and M_2 is called the residual error and is given by

$$m_p = abs(M_1 - M_2) \qquad (21)$$

where abs denotes the absolute value of the argument in bracket.

Finally, the fitting error threshold depends on this residual error and this fitting error,β is given by

$$\beta = \frac{3.5}{[geomean(m_p)]^{1/3.5}(W-1)} \qquad (22)$$

where geomean (m_p) is the geometric mean of the m_p and *W* as described above is the number of wideband signals impinging on the sensor array.

V. SIMULATION RESULTS

This section shows the performance of W-CMSR of the proposed method and compares it with the previous method. In the simulation, sensor array of seven sensors is taken having half wavelength as the spacing between the sensors. Sedumi is used as the optimization tool to solve the problem in (13). The angular grid of [-90^o $+90^o$] is taken in all the experiments with 1^o interval between them.

It can be seen in the fig. 5. that both the method are unable to estimate the direction of incoming signals. Both the method are only able to estimate the direction of incoming signal arriving at angle of 20^0 with the bias of 1^0.

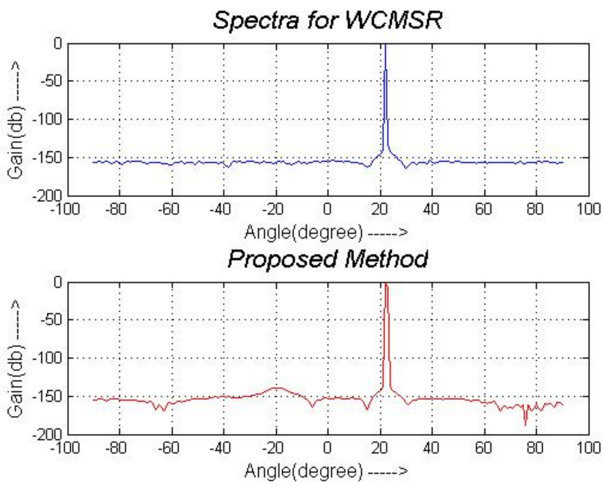

Fig. 5. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 25^0

It is clear from fig. 6. that the proposed method is able to detect the incoming signals from angle of 20^0 and 26^o whereas the previous method is unable to detect the direction of incoming signals and is only able to detect the signal incoming from 20^0.

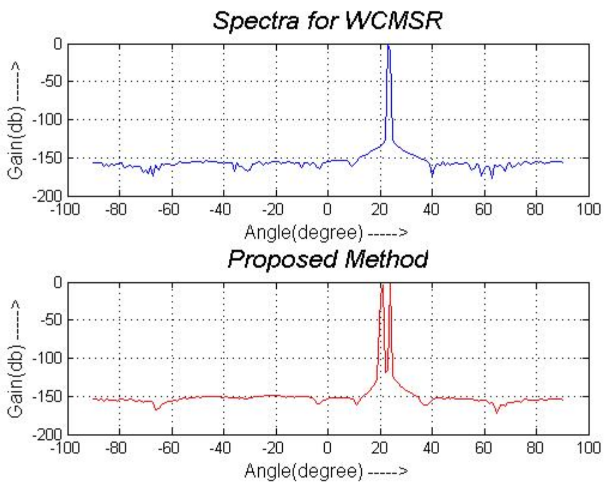

Fig. 6. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 26^0

Similar to the previous case. It is clear from fig. 7. and fig. 8. that the proposed method is able to detect the incoming signals from angle of 20^0 and 27^o or 20^0 and 28^o respectively whereas the previous method is unable to detect the direction of incoming signals and is only able to detect the signal incoming from 20^0.

Fig. 9. and Fig. 10. shows that both the method are able to detect the incoming signal when the angular resolution is greater than 8^0.

It can be seen from all the experiment conducted that the angular resolution of the proposed method is better than that of the previous method.

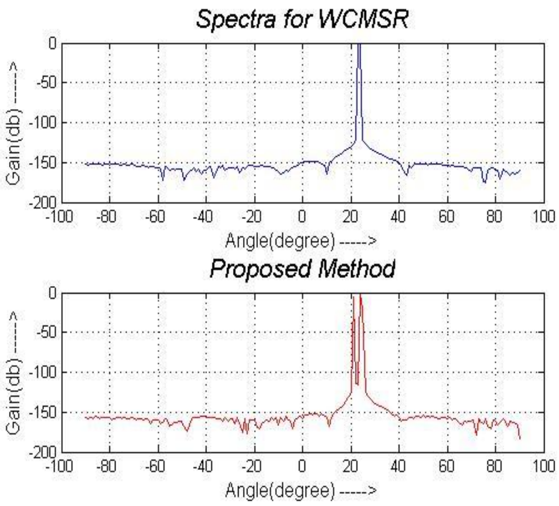

Fig. 7. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 27^0

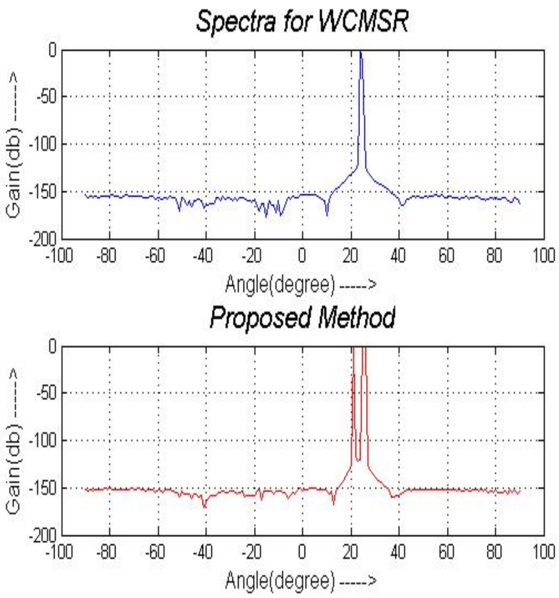

Fig. 8. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 28^0

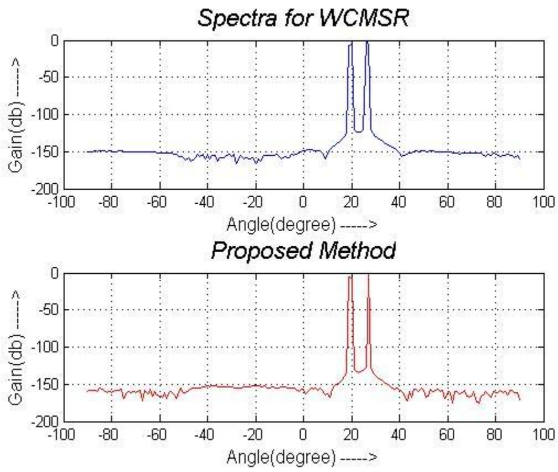

Fig. 9. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 29^0

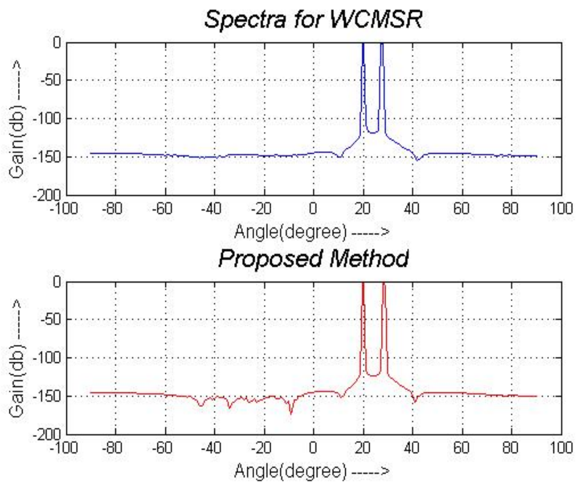

Fig. 10. Spectra of proposed W-CMSR when DOA of signals are 20^0 and 30^0

VI. CONCLUSION

An improved W-CMSR for wideband direction of arrival estimation is proposed in this paper. Simulation result shows that the new value of fitting error threshold not only improves the angular resolution of the estimation but also improves the accuracy of the estimation method. It has been further shown that the computation complexity of the proposed method is better than that of the conventional one.

VII. REFERENCES

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